Hong Kong Mathematics Olympiad (1991 – 92)
Heat Event (Individual)
香港数学竞赛 (1991 – 92)
初赛项目 (个人)

1. If $(\log_{10} x)^4 - 3(\log_{10} x)^2 - 4 = 0$ and $x > 1$, find x.
若 $(\log_{10} x)^4 - 3(\log_{10} x)^2 - 4 = 0$，且 $x > 1$，求 x。

2. If $\begin{cases} 28x + 15y = 19xy \\ 18x - 21y = 2xy \end{cases}$ and $xy \neq 0$, find x.
若 $\begin{cases} 28x + 15y = 19xy \\ 18x - 21y = 2xy \end{cases}$，且 $xy \neq 0$，求 x。

3. An integer a lying between 0 and 9 inclusive is randomly selected. It is known that the probability that the equation $x^2 - ax + 3 = 0$ has no real root is $\dfrac{p}{10}$, find p.
由 0 至 9 之中随机取一整数 a，已知方程 $x^2 - ax + 3 = 0$ 无实根的概率为 $\dfrac{p}{10}$，求 p。

4. $x°$ is an acute angle satisfying $\dfrac{1}{2}\cos x° \geq \dfrac{1}{2}(5 - \cos x°) - 2$. Determine the largest possible value of x.
$x°$ 为一满足 $\dfrac{1}{2}\cos x° \geq \dfrac{1}{2}(5 - \cos x°) - 2$ 的锐角，求 x 的最大值。

5. Let $f(x)$ be the highest common factor of $x^4 + 64$ and $x^3 + 6x^2 + 16x + 16$, find $f(2)$.
设 $f(x)$ 为 $x^4 + 64$ 和 $x^3 + 6x^2 + 16x + 16$ 的最大公因式，求 $f(2)$。

6. A fruit merchant divides a large lot of oranges into four classes: A, B, C, D. The number of oranges in class A and class B doubles that in class C while the number of oranges in class B and class D doubles that in class A. If 7 oranges from class B are upgraded to class A, class A will then contain twice as many oranges as class B. It is known that one of the four classes contains 54 oranges. Determine which one class it belongs to.
果商把一堆橙分成 A、B、C、D 四个等级。A 级和 B 级橙的数目合起来是 C 级的两倍；B 级和 D 级橙的数目合起来是 A 级的两倍。若将 B 级橙中的 7 个升格为 A 级，则 A 级的橙数便成为 B 级的两倍。已知 A、B、C、D 四级橙中，其中某级有橙 54 个，问这是哪一级？

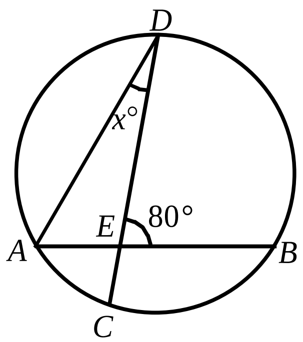
7. Given that n is a positive integer, find **ALL** the real roots of $x^{2^n} - 10^{2^n} = 0$.
已知 n 为一正整数，求 $x^{2^n} - 10^{2^n} = 0$ 的所有实根。

8. If n is an integer randomly selected from 1 to 100, and the probability that the unit digit of 5678^n is greater than 3 is $\frac{3}{x}$, find x.

若 n 是从 1 至 100 中随意选取的整数，且 5678^n 的个位数大于 3 的概率是 $\frac{3}{x}$，求 x。

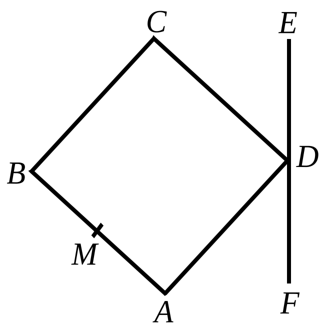
9. In ΔABC, $AB = 8$ cm, $BC = 6$ cm and $\angle ABC = 90°$. If the bisector of $\angle ACB$ cuts AB at R and $CR = 3\sqrt{a}$ cm, find a.
在 ΔABC 中，$AB = 8$ cm、$BC = 6$ cm、$\angle ABC = 90°$，若 $\angle ACB$ 的角平分线与 AB 交于 R，且 $CR = 3\sqrt{a}$ cm，求 a。

10. In figure 1, arc BD is 4 times the arc AC, $\angle DEB = 80°$ and $\angle ADC = x°$, find x.
在图 1 中，弧 BC 的长度是弧 AC 的 4 倍，$\angle DEB = 80°$ 及 $\angle ADC = x°$，求 x。



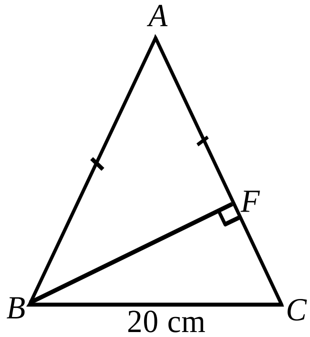
(Figure 1) (图 1)

11. In figure 2, $ABCD$ is a square. EDF is a straight line. M is the mid-point of AB. If the distances of A, M and C from the line EF are 5 cm, 11 cm and x cm respectively, find x.
在图 2 中，$ABCD$ 是一正方形，EDF 是一直线，M 是 AB 的中点。若 A、M 和 C 到直线 EF 的距离依次为 5 cm、11 cm 和 x cm，求 x。



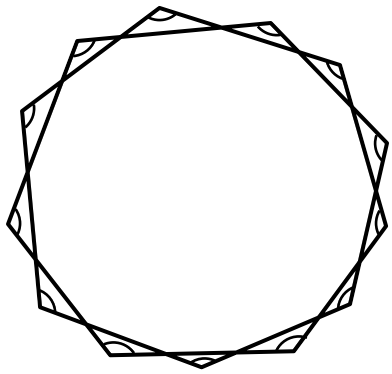
(Figure 2) (图 2)

12. In figure 3, $AB = AC = 2BC$ and $BC = 20$ cm. If BF is perpendicular to AC and $AF = x$ cm, find x.
在图 3 中，$AB = AC = 2BC$ 及 $BC = 20$ cm。 若 BF 垂直于 AC，且 $AF = x$ cm，求 x。

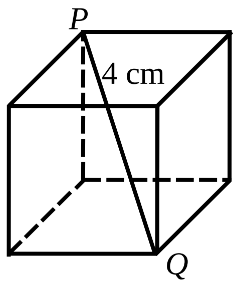


(Figure 3) (图 3)

13. Figure 4 shows a figure obtained by producing the sides of a 13-sided polygon. If the sum of the marked angles is $n°$, find n.
图 4 是延长一个 13 边形的边所构成的图形。若图中标示的角的和是 $n°$，求 n。


(Figure 4) (图 4)

14. In figure 5, PQ is a diagonal of the cube. If $PQ = 4$ cm and the total surface area of the cube is x cm^2, find x.
在图 5 中，PQ 为一正方 体的对角线。若 $PQ = 4$ cm，且这正方体的总表面面积为 x cm^2，求 x。



(Figure 5) (图 5)

15. If $(3x-1)^7 = a_1x^7 + a_2x^6 + a_3x^5 + \cdots + a_8$, find the value of $a_1 + a_2 + a_3 + \cdots + a_8$.
若 $(3x-1)^7 = a_1x^7 + a_2x^6 + a_3x^5 + \cdots + a_8$，求 $a_1 + a_2 + a_3 + \cdots + a_8$ 的值。

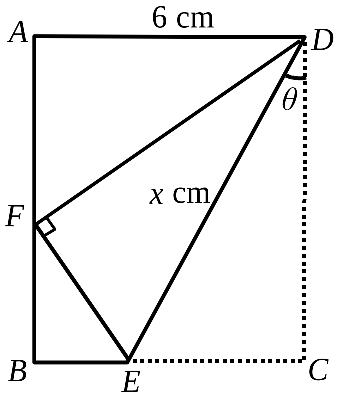
16. $A(1, 1)$, $B(a, 0)$ and $C(1, a)$ are the vertices of the triangle ABC. Find the value of a if the area of

ΔABC is 2 square units and $a>0$.
$A(1,1)$、$B(a,0)$、$C(1,a)$ 是三角形 ABC 的顶点，若 ΔABC 的面积是 2 平方单位，且 $a>0$，求 a。

17. If $N=2^{12}\times 5^{8}$, find the number of digits of N.
若 $N=2^{12}\times 5^{8}$，N 是一个多少位的数字？

18. If $a:b=3:4$ and $a:c=2:5$, find the value of $\dfrac{ac}{a^2+b^2}$.
若 $a:b=3:4$ 及 $a:c=2:5$，求 $\dfrac{ac}{a^2+b^2}$ 的值。

19. A rectangular piece of paper of width 6 cm is folded such that one corner touches the opposite side as shown in figure 6. If $\theta=30°$ and $DE=x$ cm, find x.
一张阔 6 cm 的长方形纸按图 6 所示对折，使得一角与对边接触。若 θ 为 30°，且 $DE=x$ cm，求 x。



(Figure 6) (图 6)

20. If $\sin x+\cos x=\dfrac{1}{5}$ and $0\le x\le \pi$, find $\tan x$.
若 $\sin x+\cos x=\dfrac{1}{5}$，且 $0\le x\le \pi$，求 $\tan x$。